

1 Description of Maple Files

The first file NkCalculationNew.mw It calculates $\mathcal{N}_{1/2}[k]$ for specified R, ν for $1 \leq k \leq k_m$ where $k_m = \max \left\{ \sqrt{\frac{3}{\nu}} R, 6\nu^{-1/2} \right\}$. though in the program, we specifically set $\Lambda = \frac{1}{2}$. This is based on the electronic supplementary material §2. The notation is completely consistent. The calculations also looks at relative sizes of

1. $I_2 I_3$ and $I_2 I_3 - I_1 I_4$,
2. $B_2 I_1$ and $B_2 I_1 - B_1 I_3$,
3. $B_2 I_2$ and $B_2 I_2 - B_1 I_4$ as well as
4. $\lambda_1 e^{-\alpha}(B_2 I_1 - B_1 I_3) - \lambda_2 e^{\alpha}(B_2 I_2 - B_1 I_4)$

by checking in each case whether the ratio of the subtracted quantity divided by the first object was less than $\frac{1}{10}$ and if it was take the log base 10 of the reciprocal of that number and sum all four of such numbers to give us an upper bound of extra loss of accuracy beyond the atmost 4 digit loss of accuracy due to four addition/subtraction operations. Here we are assuming that Maple calculates the integrals I_j to the precision it claims. With this, $\mathcal{N}_{1/2}[k]$ calculations are confirmed to at least 12 digit accuracy. The file NkCalculationNew.mw was used to generate the 12 different data files NkkMapleRhalfR20nup1.dat, NkkMapleIhalfR20nup1.dat, NkkMapleRhalfR20nup05.dat, NkkMapleIhalfR20nup05.dat, NkkMapleRhalfR50nup1.dat, NkkMapleIhalfR50nup1.dat, NkkMapleRhalfR50nup05.dat, NkkMapleIhalfR50nup05.dat, NkkMapleRhalfR100nup1.dat, NkkMapleIhalfR100nup1.dat, NkkMapleRhalfR100nup05.dat, NkkMapleIhalfR100nup05.dat. Part of the naming convention is similar, for instance, the part of the name R20nup1 corresponds to $R = 20$, $\nu = 0.1$. This is preceded either by *Rhalf* or *Ihalf* corresponding to real and imaginary parts of $\mathcal{N}_{1/2}[k]$, the half in the name is a reminder that we used $\Lambda = \frac{1}{2}$ in the $\mathcal{N}_0[k]$ formula.

The second set of files pertain to using steady state calculation data as potential quasi-solution and checking that the conditions of Theorem 1 are indeed satisfied. The file also has determination of bifurcation points for $k_0 = 1$ and $k_0 = 2$ solutions from the $H = 0$ state, and plotting the profiles based on quasi-solution. The 12 files are named dimitriR20nup1first.mw, dimitriR20nup05first.mw, dimitriR50nup1first.mw, dimitriR50nup05first.mw, dimitriR100nup1first.mw, dimitriR100nup05first.mw, dimitriR20nup1second.mw, dimitriR20nup05second.mw, dimitriR50nup1second.mw, dimitriR50nup05second.mw, dimitriR100nup1second.mw, dimitriR100nup05second.mw.

The first, and second in the names correspond to $k_0 = 1, 2$ branches respectively. the part of the file name ..R20nup1.. corresponds to $R = 20$, $\nu = 0.1$ with Λ over a range which is clear in the files themselves. Similar convention was used for all other files with R and ν values given. Comments are given in the dimitriR20nup1first.mw to explain in terms of material in the text relating to theorem 1. Note, we did not put extensive comments on the other dimitri files, since the same logic is being used.

The dimitri.. files rely on corresponding input data files that were generated from fortran programs are adatanewcnR20nup1Lp302t2p0first.dat, adatanewcnR20nup05Lp151t1p2first.dat, adatanewcnR50nup1Lp123t2p0first.dat, adatanewcnR50nup05Lp07t1p2first.dat, adatanewcnR100nup1Lp065t2p0first.dat, adatanewcnR100nup05Lp032t2p0first.dat, adatanewcnR20nup1Lp121t2p2second.dat, adatanewcnR20nup05Lp602t1p3second.dat, adatanewcnR50nup1Lp51t2p0second.dat, adatanewcnR50nup05Lp25t2p0second.dat, adatanewcnR100nup1Lp3t2p0second.dat and adatanewcnR100nup05Lp136t2p0second.dat .

The convention used is the same with respect to R, ν , first and second appearing in part of each of the names. Additionally, the name has the part *Lp.t..* which denotes the range of Λ values in steps of 0.001. For instance, adatanewcnR20nup1Lp302t2p0first.dat contains all the

fortran file data for $R = 20$, $\nu = 0.1$ for $\Lambda = 0.302$ to $\Lambda = 2.0$ in steps of 0.001 for the $k_0 = 1$ branch. The smallest value reported is close to the $H = 0$ bifurcation point. We note that the data files are in stacks of 257 data points, where the first line denotes C_0 as the first entry (you can ignore the second) and the remaining 256 lines each with two columns indicate real and imaginary parts of $H_0(k)$. 256 modes is an overkill. In most cases 20 mode truncation for quasi-solution was good enough to get all conditions of Theorem 1 to hold. In some extreme cases, we needed 40 modes or more.

Since how the quasi-solution candidates are generated is immaterial to the proof as long as they satisfy the conditions of the Theorem, we do not include the fortran code that generated the files adatanew..